Physica A 492 (2018) 1340–1351

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Social outcomes due to the interplay between language competition and ideology struggle

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HIGHLIGHTS

- A Lotka–Volterra-type model is proposed to study the interplay between language competition and ideology struggle.
- Bilingualism is sustained only in a segregated society with a bilingual group coexisting with an isolated nationalistic monolingual group.
- Complete language assimilation and extinction of nationalism implies the survival of the language spoken by former nationalists.
- Linguistic segregation, nationalism and isolation are negative social outcomes of ideology struggle when two monolingual groups survive in the equilibrium.
- Nationalism might survive even when ideology struggle favours cultural assimilation.

ARTICLE INFO

Article history:
Received 14 March 2017
Received in revised form 20 September 2017
Available online 20 November 2017

Keywords:
Sociophysics
Language competition
Ideology struggle
Nationalism
Population dynamics

ABSTRACT

I study the interplay between language competition and ideology struggle in a country where there are two competing languages. Language transition is governed by a three-state model similar to Minett–Wang (2008) and Heinsalu et al. (2014). In this class of models, I further assume that among monolinguals of one of the competing languages there is an ideology struggle between assimilationist individuals who accept to deal with foreign language speakers and nationalist individuals who oppose any form of foreign culture. Ideology transition follows a two-state model as in Abrams–Strogatz (2003). Depending on both ideology and language status, the possible equilibria show that when nationalism is introduced in the language competition model, complete assimilation might take place and one language disappears with the entire population becoming monolingual. On the other hand, when bilingualism emerges, it is associated with a segregated society with a bilingual group surviving in the long run together with an isolated monolingual group entirely composed of nationalist individuals. Another kind of segregation might also emerge in the equilibrium, in which two monolingual groups survive in the long run, one of them entirely composed of nationalists. In the latter case, both linguistic segregation and isolation are the negative social outcomes of ideology struggle.

1. Introduction

The issues of opinion formation, population dynamics, ideology struggle, religion and church growth, language competition and the dynamics of language death have attracted the interest of many physicists, particularly those applying the tools

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https://doi.org/10.1016/j.physa.2017.11.061
0378-4371/

of statistical mechanics and complex systems [1–16]. Such population models deal with some entity (e.g.: language) being diffused through a population. Two frameworks often found in the literature to model social diffusion are predator–prey models and epidemic models. One application of the former is in language competition where the speaker of one language is responsible for its spread through contact with others speaking a competing language. In [1] a predator–prey model is used to study the interaction between a bilingual and a majority monolingual population. The model is further investigated in [4] with applications to the study of minority languages in Wales (Welsh), Brittany (Breton), Ireland (Irish) and Scotland (Gaelic). An extension of the work in [4] takes into account a second smaller monolingual population and includes language planning as a factor affecting language-group interaction and evolution (see Ref. [5]). Language acquisition planning measures are put in place whenever the number of speakers of the disadvantaged language is below some threshold.

In contrast, epidemic models are better used in social diffusion processes where the ability or enthusiasm to spread some entity (e.g.: religious belief) has limited duration. The simplest epidemic models consider three categories of people. Susceptibles (S) are those individuals who become infected due to contact with an infective (I). After some period of time, the individuals in the latter group become part of the removed (R) group, i.e., out of the infectious state due to isolation, death or immunization after recovering. The model in [2] consists of a SIR-type epidemic model to study church growth, in which susceptibles (S) are non-believers, not belonging to the church, who might become members of the church due to contact with enthusiast believers (I). The latter are members of the church who are active in spreading the faith. After some period of time, enthusiast believers remain church members but, although still part of the community of believers, they become inactive in the role of spreading the faith. This requires a third category of people in such population models, the removed (R), who are inactive believers, removed from the spreading process. Such framework is extended in [3] where the SIR-model also takes into account the effects of birth, death, new church members who lack enthusiasm, adult reversion, reconversion and the role of hardened unbelievers in church growth. Another application of epidemic models can be seen in [6] to study growth and decline in the membership of political parties.

Despite the connection between this wide variety of social issues, studies dealing with their combined effects on society are still relatively rare. In this paper, a Lotka–Volterra-type model is used to investigate the interplay between language competition and ideology struggle in a country where there are two languages competing against each other for speakers as in [7–15]. Bilingual individuals are taken into account as in [9–11] and I introduce the novelty that monolinguals of one competing language face an ideological struggle between a group of nationalists who oppose foreign speakers and a group whose members welcome the latter and might have an interest in learning their culture and language. Such kind of ideology struggle was studied in [17] in a different framework using evolutionary game theory. The relevance and motivation to integrate the evolution of ideology and language into the same model is discussed in Ref. [18]. According to the latter, language and religion have been the two most important cultural infrastructures serving as bases for national differentiation and modern demands of autonomy in Europe.

A class of mean-field approximation models dealing with language competition attracted the interest of physicists particularly after the work of [7]. The latter adopts an approach similar to [1] and provides a two-state model for language shift, where homogeneous mixing is assumed and two languages compete with each other for speakers. Languages are assumed as fixed entities, i.e., grammar, syntax and other structural properties of language do not evolve over time. The probability of an individual shifting from one language to the other is proportional to the attracting language perceived social status and its population size of speakers. Two drawbacks of the model are the assumption that no individual is bilingual and the absence of an equilibrium in which languages can coexist.

A three-state model, with the explicit introduction of a group of bilinguals was introduced in [9]. As long as a minimum threshold for language similarity is satisfied, bilingualism becomes a stable equilibrium in which the bilingual group survives in the long run together with the group of monolingual individuals speaking the high-status language. Criticism regarding such a result was addressed in [10] due to the possibility of direct language shift between monolingual speakers. Such transitions would be very unlike in the real world as those would involve the simultaneous loss of one language and the acquisition of the competing one, i.e., children would acquire a different language from the one of their parents or monolingual adults learning a new language would simultaneously forget their native language. In order to overcome this issue, the work in [10] extends the model in [7], accounting for a bilingual group as in [9], and introduces a distinction between horizontal and vertical transmission of language. The mechanism of language transmission is more restrictive than that of [9] as direct transitions between monolingual groups are not allowed. Despite being more realistic, the base model of [10] shares the same prediction as in [7], i.e., one of the two competing languages will eventually become extinct.

A successful attempt to model language competition in which both the vertical and horizontal mechanisms of language transmission are respected and bilingualism can be sustained without any form of intervention is proposed in [11]. In their Lotka–Volterra-type model, bilinguals play a role in the language transition probabilities and survival of bilingual individuals, who take over the entire population, is an asymptotically stable stationary point if bilinguals are sufficiently regarded by monolinguals as speakers of the other competing language.

Thus, the model in [7] has been successively improved in the literature over time until bilingualism could be sustained in the long run and a low-status language could not necessarily be doomed. A concise literature review on language competition dynamics, including mean-field approximation models, can be found in [14]. In the latter, the main concern is whether competition leads to the coexistence of languages or not. Overall, bilingualism is a positively regarded outcome in the literature given it avoids the prevalence of just one language taking over the entire population.

In this paper, I introduce the interplay between the issues of ideology struggle and language competition in this class of models, showing that in such a case an equilibrium with the survival of bilingualism in the long run might not be a

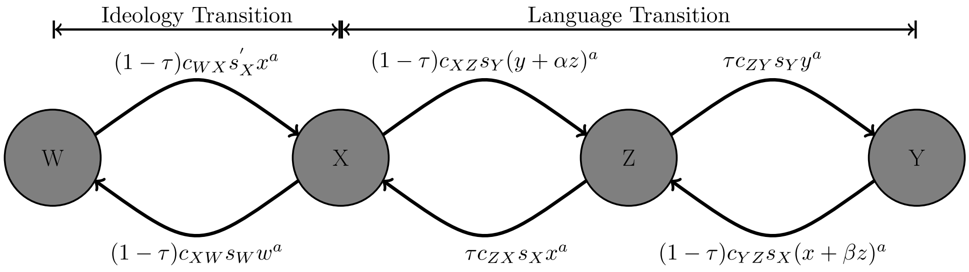

Fig. 1. General scheme for ideology and language transition rates.

desired result from a social point of view. The connection between language competition and ideology struggle in the same model is a natural step given that Lotka–Volterra-type models were previously used to predict the evolution of competing ideologies. In [16], an ideological competition in a country with growing population is addressed. The evolution of the aggregate population follows the Verhulst law and the number of individuals belonging to each ideological group follows a Lotka–Volterra equation.

Regarding the contributions of this paper, depending on both ideology and language status, the possible equilibria show that: (i) when nationalism is introduced in the language competition model, complete assimilation might take place and one language disappears with the entire population becoming monolingual (as in [7,10]). Nationalism becomes extinct; (ii) on the other hand, when bilingualism emerges, it is associated with a segregated society with a bilingual group surviving in the long run together with an isolated monolingual group entirely composed of nationalist individuals. Nationalism and segregation could pave the way to violence despite bilingual individuals being able to speak the language of nationalists; (iii) another kind of segregation, with two monolingual groups surviving in the long run, one of them entirely composed of nationalists might also emerge. When this happens, both linguistic segregation and isolation are the negative social outcomes of ideology struggle.

The model can be used for example to address how nationalism and assimilation evolve among natives and immigrants, respectively, as in the evolutionary game presented in [17]. In the latter, depending on the parameters of the model, a dynamics of Lotka–Volterra type can be obtained with nationalism and immigrant assimilation oscillating over time. Instead, in the present work, a non-oscillatory neutrally stable state with nationalism and segregation can be found.

The remainder of the paper is organized as follows: in Section 2, the model unifying the impact of both ideology and language competition on society is presented. Section 3 discusses the results. Section 4 addresses a particular extension in which nationalists are less likely to change their ideology while assimilationists are easier to be convinced to join nationalism. Section 5 concludes suggesting further research.

2. Model

I consider a four-state model where the population living in a country can be divided into four groups denoted X, Y, Z and W, with three types of speakers: a proportion $y \in Y$ of monolingual individuals who only speak language B, a proportion $z \in Z$ of bilingual individuals who speak both languages A and B and the remainder of individuals are monolinguals who only speak language A, the latter being split into two groups, with a proportion $x \in X$ of individuals who are open-minded (assimilationists) regarding foreign culture and foreign language while a proportion $w \in W$ are of individuals who share a nationalistic ideology and are against any form of foreign culture, where $x + y + z + w = 1$. Fig. 1 presents the four groups as well as the transition rates and possible transitions between groups. Note that by default, the model assumes that nationalism only exists among individuals belonging to W, thus, individuals in X, Y and Z do not discriminate.

As in [10], with probability τ (mortality rate), adults are replaced by children and both language and ideology follow the vertical model of language transmission discussed in [10,11], i.e., a child of a monolingual individual necessarily acquires her parent's language as her mother tongue, while a child of a bilingual individual might either acquire only one or both languages. Regarding ideology, I also assume that the first values and social norms acquired by a child are those of her parent, thus a child of an individual belonging to group W initially grows up with nationalistic values while children of individuals belonging to the other groups do not discriminate anyone. Possible transitions regarding the vertical transmission mechanism are thus $X \to X, Y \to Y, Z \to Z, W \to W, Z \to X, Z \to Y$. As in [10,11], there is no direct transition between groups X and Y because this would involve a child being unable to communicate with her parent.

On the other hand, with probability $1 - \tau$, the model follows the horizontal mechanism of language transmission in [10,11], i.e., a bilingual adult remains bilingual over his entire life while a monolingual adult might remain monolingual or learn a second language. With regard to ideology, I assume that over life A-monolingual individuals might remain nationalist or change their ideology and become open-minded (transitions $W \to W$ and $W \to X$). The same is valid regarding individuals in group X (transitions $X \to X$ and $X \to W$). This is in line with ideology struggle in [16,17]. Thus, possible transitions regarding horizontal transmission are $X \to X, Y \to Y, Z \to Z, W \to W, X \to Z, X \to W, W \to X$ and $Y \to Z$.

Following the above rules, transition from monolingualism to bilingualism necessarily takes place through horizontal transmission and transition in the opposite direction is made through vertical transmission. The model assumes no direct transition between W and Z because only in the rarest case a nationalist would be interested in learning a foreign culture and language. Following the most natural path, a nationalist would first change his ideology, start welcoming monolinguals of the competing language and their culture and only then could develop an interest in learning language B ($W \to X \to Z$).

The vertical and horizontal language transmission mechanisms found in [10,11] can also be seen in the stock-flow structure of the operational research model applied to language group interaction in [19]. In the latter, vertical transmission of language is related to how infants (0–3 years cohort) are raised: infants born to monolingual parents are raised monolingual but might be raised either monolingual or bilingual if born to bilingual parents. Monolingual children (4–18 years cohort) and monolingual adults (19+ years cohort) might then be taught at school or learn in adult programmes or at home a second language, which is similar to the horizontal transmission mechanism.

As can be seen in Fig. 1, transition probabilities from the bilingual group Z to a monolingual group are proportional to a constant c_{Zi}; $i = \{X, Y\}$, to the mortality rate τ, to the status of the attracting language $s_i \in [0, 1]$; $i = \{X, Y\}$ and to the size of the attracting group. As usually assumed in the literature [7,9,10], the relation between the status of the competing languages satisfies $s_X + s_Y = 1$. On the other hand, transition probabilities in the opposite direction are proportional to a constant c_{iZ}; $i = \{X, Y\}$, to $1 - \tau$, to the status of the attracting language and to the size of the attracting group, in this case either $x + \beta z$ or $y + \alpha z$. Similar to [11], $\alpha \in (0, 1)$ and $\beta \in (0, 1)$ are the importance of bilinguals as representatives of the B and A languages to monolinguals in groups X and Y, respectively. Constant parameters c_{Zi} and c_{iZ} reflect sociolinguistic factors [11] such as the propensity of individuals to learn a new language based on their existing linguistic skills or the provision of language resources for children [10]. Hence, language transition is governed by a three-state model as in [10] with the extension introduced in [11] such that bilinguals might influence the transition probabilities.

Regarding the interplay between the two ideologies, the model is similar to [7], i.e., transitions occur between two-states, the nationalistic ideology and the assimilationist one. Transition probabilities are proportional to a constant c_i; $i = \{WX, XW\}$, to $1 - \tau$, to the status of each ideology (either $s'_X \in [0, 1]$ or $s_W = 1 - s'_X$) and to the size of the attracting ideological group. Constants c_{XW} and c_{WX} play a similar role as the constants in the model of [16], i.e., larger values are related to a more intense ideology conversion.

Similarly to other predator–prey models applied to language or ideology competition in the literature, it should be emphasized that whenever an individual shifts from group i to group j, the latter acts as the active population or the predator-analogue, while the former group is the passive or prey-analogue, as described in [4]. Based on the discussed assumptions, the evolution of the share of each group in the total population is given by the following system of non-linear ordinary differential equations:

$$\dot{x} = \tau c_{ZX} s_X z x^a - (1-\tau)\left[c_{XZ}(1-s_X)x(y+\alpha z)^a + c_{XW}(1-s'_X)xw^a - c_{WX}s'_X w x^a\right] \tag{1}$$

$$\dot{y} = \tau c_{ZY}(1-s_X)zy^a - (1-\tau)c_{YZ}s_X y(x+\beta z)^a \tag{2}$$

$$\dot{w} = (1-\tau)\left[c_{XW}(1-s'_X)xw^a - c_{WX}s'_X w x^a\right] \tag{3}$$

where $\dot{x} = dx/d\phi$, $\dot{y} = dy/d\phi$, $\dot{w} = dw/d\phi$, $\dot{z} = dz/d\phi = -\dot{x} - \dot{y} - \dot{w}$ and $d\phi$ denotes the time interval. The phase space is given by the unit tetrahedron $\Omega = \left\{\theta \in [0, 1]^3 : x + y + w \le 1\right\}$. Despite the focus of sociophysics models of language competition and ideology struggle on mean-field pairwise interactions as the main trigger for state transitions, some interesting demographic dynamics can be contemplated by the model in Fig. 1 such as the typical three-generations pattern of complete assimilation of immigrants into the native culture found in the US and discussed in [20,21]: a monolingual immigrant arriving in the country makes an effort to learn the native language (transition $Y \to Z$), gives birth to a child who learns both the native and the foreign language, thus still preserving the parent culture ($Z \to Z$), while that second generation individual gives birth to a child who only acquires the native language ($Z \to X$). As an adult, the latter might become nationalist ($X \to W$) or not. In the opposite direction, native Americans might have an interest or not to learn Spanish, particularly in places such as Miami, transitions $X \to X$ or $X \to Z$, while transitions $Z \to Y$ would be rare in the particular case of the US.

Following [7,9], all constants c_i; $i = \{ZX, XZ, XW, WX, ZY, YZ\}$ are set equal, $c_i = c$. Also, as in [10,11], I set $a = 1$, i.e., the attractiveness of a language (ideology) increases linearly with its proportion of speakers (followers). In the remainder of the paper, I will focus on the case where vertical and horizontal transmission are equally likely, $\tau = 0.5$. Thus, taking into account that $z = 1 - x - y - w$, Eqs. (1)–(3) become:

$$\dot{x} = 0.5cx\left\{s_X(1-x-y-w) - \left[(1-s_X)(y+\alpha(1-x-y-w)) + (1-2s'_X)w\right]\right\} \tag{4}$$

$$\dot{y} = 0.5cy\left[(1-s_X)(1-x-y-w) - s_X(x+\beta(1-x-y-w))\right] \tag{5}$$

$$\dot{w} = 0.5cxw(1-2s'_X). \tag{6}$$

In Eqs. (4)–(6), let a time rescaling take place in the system in which the time interval $d\phi$ is replaced by another time interval dt where $d\phi = (0.5c)^{-1}dt$. Thus, $\dot{x} = \frac{dx}{d\phi} = \frac{dx}{dt}(0.5c)$, leading the constant $0.5c$ to be set to 1 in the right hand side of (4). The same time rescaling applies to both (5) and (6), leading to:

$$\dot{x} = x\left[(s_X - \alpha(1-s_X))(1-x-y-w) - (1-s_X)y - (1-2s'_X)w\right] \tag{7}$$

$$\dot{y} = y\left[(1-s_X-\beta s_X)(1-x-y-w) - xs_X\right] \tag{8}$$

$$\dot{w} = xw(1-2s'_X) \tag{9}$$

where $\dot{x} = dx/dt$, $\dot{y} = dy/dt$ and $\dot{w} = dw/dt$. The system in (7)–(9) has the following isolated fixed points: $(1, 0, 0)$ and $(\bar{x}, \bar{y}, 0)$, where $\bar{x} = \frac{(1-s_X)[1-s_X(1+\beta)]}{g(s_X)}$, $\bar{y} = \frac{s_X[s_X-\alpha(1-s_X)]}{g(s_X)}$ and $g(s_X) = 1 - s_X(1-s_X)(1+\alpha+\beta) > 0,\ \forall s_X \in [0, 1]$. Moreover, $(\bar{x}, \bar{y}, 0) \in \Omega$ if $\alpha(1+\alpha)^{-1} < s_X < (1+\beta)^{-1}$. The following sets of fixed points are also in Ω: $(0, 0, w)$ and $(0, y, 1-y)$. For the remainder of the paper, I will call the status of language A low, intermediate or high if s_X satisfies $0 < s_X < \alpha(1+\alpha)^{-1}$, $\alpha(1+\alpha)^{-1} < s_X < (1+\beta)^{-1}$ or $(1+\beta)^{-1} < s_X < 1$, respectively.

Regarding the dynamics and the topology in the interior of Ω, $\dot{w} \gtrless 0$ if $s'_X \lessgtr 1/2$. $\dot{y} = 0$ at any point located on the plane $\mathcal{P}_Y : x = \frac{(1-y-w)(1-s_X-s_X\beta)}{1-s_X\beta}$. $\mathcal{P}_Y$ crosses the boundaries of Ω at the vertices $(0, 1, 0)$, $(0, 0, 1)$ and at $\left(\frac{1-s_X-s_X\beta}{1-s_X\beta}, 0, 0\right)$, thus $\mathcal{P}_Y$ crosses the interior of Ω if $s_X < (1+\beta)^{-1}$. Any trajectory starting in the interior of Ω and above (resp. below) $\mathcal{P}_Y$ satisfies $\dot{y} < 0$ (resp. $\dot{y} > 0$). If $\mathcal{P}_Y \bigcap \Omega_{\text{interior}} = \emptyset$, $\dot{y} < 0$ for any trajectory starting in the interior of Ω. Also, $\dot{x} = 0$ at any point located on the plane $\mathcal{P}_X : x = \frac{(1-y-w)(s_X-\alpha(1-s_X))-(1-s_X)y-(1-2s'_X)w}{s_X-\alpha(1-s_X)}$. When $s'_X > 1/2$, $\mathcal{P}_X$ always crosses the interior of Ω, crossing the boundaries of Ω at the vertex $(1, 0, 0)$ and at $\left(0, \frac{2s'_X-1}{2s'_X-s_X}, \frac{1-s_X}{2s'_X-s_X}\right)$ and $\left(0, 0, \frac{s_X-\alpha(1-s_X)}{(s_X-\alpha(1-s_X))+(1-2s'_X)}\right)$, if $s_X < \alpha(1+\alpha)^{-1}$. Otherwise, if $s'_X > 1/2$ and $s_X > \alpha(1+\alpha)^{-1}$ instead, $\mathcal{P}_X$ crosses the boundaries of Ω at the vertex $(1, 0, 0)$ and at $\left(0, \frac{2s'_X-1}{2s'_X-s_X}, \frac{1-s_X}{2s'_X-s_X}\right)$ and $\left(0, \frac{s_X-\alpha(1-s_X)}{1-\alpha(1-s_X)}, 0\right)$. On the other hand, when $s'_X < 1/2$, $\mathcal{P}_X$ crosses the boundaries of Ω at the vertex $(1, 0, 0)$ and at $\left(0, \frac{s_X-\alpha(1-s_X)}{1-\alpha(1-s_X)}, 0\right)$ and $\left(0, 0, \frac{s_X-\alpha(1-s_X)}{(s_X-\alpha(1-s_X))+(1-2s'_X)}\right)$, thus crossing the interior of Ω whenever $s_X > \alpha(1+\alpha)^{-1}$ is satisfied. Whenever $\mathcal{P}_X \bigcap \Omega_{\text{interior}} \neq \emptyset$, any trajectory starting in the interior of Ω and above (resp. below) $\mathcal{P}_X$ satisfies $(-1)^i\dot{x} < 0$ (resp. $(-1)^i\dot{x} > 0$), where $i = 0$ if $s_X > \alpha(1+\alpha)^{-1}$, otherwise $i = 1$. Finally, if $\mathcal{P}_X \bigcap \Omega_{\text{interior}} = \emptyset$, $\dot{x} < 0$ for any trajectory starting in the interior of Ω. The dynamics and topology described above will be useful in Section 3 where simulations showing the phase space will be presented.

Proposition 1. *There are six possible cases regarding language and ideology status, for which the system of non-linear differential equations in (7)–(9) has the equilibria presented in Table 1.*

Proof. In order to study the stability of the fixed points, the Jacobian matrix θ is given by:

$$\frac{\partial \dot{x}}{\partial x} = \left[(s_X - \alpha(1-s_X))(1-2x-y-w) - (1-s_X)y - (1-2s'_X)w\right]$$

$$\frac{\partial \dot{x}}{\partial y} = -x\left[1-\alpha(1-s_X)\right]$$

$$\frac{\partial \dot{x}}{\partial w} = -x\left[(s_X-\alpha(1-s_X)) + (1-2s'_X)\right]$$

$$\frac{\partial \dot{y}}{\partial x} = -y(1-\beta s_X)$$

$$\frac{\partial \dot{y}}{\partial y} = \left[(1-s_X-\beta s_X)(1-x-2y-w) - xs_X\right]$$

$$\frac{\partial \dot{y}}{\partial w} = -y(1-s_X-\beta s_X)$$

$$\frac{\partial \dot{w}}{\partial x} = w(1-2s'_X)$$

$$\frac{\partial \dot{w}}{\partial y} = 0$$

$$\frac{\partial \dot{w}}{\partial w} = x(1-2s'_X).$$

The eigenvalues λ_i of θ evaluated at $(1, 0, 0)$ are $\lambda_i = \left\{-s_X + (1-s_X)\alpha;\ -s_X;\ 1-2s'_X\right\}$, which are all negative provided $s'_X > 0.5$ and $s_X > \frac{\alpha}{1+\alpha}$, i.e., a population of monolinguals of language A is an asymptotically stable state provided the status of language A is either intermediate or high and the status of the assimilationist ideology is stronger than the nationalistic one. The latter corresponds to Cases 5 and 6 in Table 1. On the other hand, at $(\bar{x}, \bar{y}, 0)$, the eigenvalues of θ are the solutions to $(\frac{\partial \dot{w}}{\partial w} - \lambda)(\lambda^2 - \text{trace}(\theta_{2X2})\lambda + \det(\theta_{2X2})) = 0$, where $\det(\theta_{2X2}) = \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{y}}{\partial y} - \frac{\partial \dot{x}}{\partial y}\frac{\partial \dot{y}}{\partial x} = -\bar{x}\bar{y}g(s_X) < 0$, leading the eigenvalues of the Jacobian matrix evaluated at $(\bar{x}, \bar{y}, 0)$ to always have mixed signs, thus the fixed point $(\bar{x}, \bar{y}, 0)$ is unstable. At the

Table 1
Possible equilibria given A-language status s_X and assimilationist ideology status s'_X.

Possible cases	Neutrally stable sets and asymptotically stable points
Case 1: low A-language status and stronger nationalism $0 < s_X < \alpha(1+\alpha)^{-1}$ and $s'_X < 1/2$	$(0, y, 1-y),\ \forall y \in [0, 1]$
Case 2: intermediate A-language status and stronger nation. $\alpha(1+\alpha)^{-1} < s_X < (1+\beta)^{-1}$ and $s'_X < 1/2$	$(0, y, 1-y),\ \forall y \in [0, 1]$
Case 3: high A-language status and stronger nationalism $(1+\beta)^{-1} < s_X < 1$ and $s'_X < 1/2$	$(0, 0, w),\ \forall w \in \left(\frac{s_X-(1-s_X)\alpha}{(s_X-(1-s_X)\alpha)+(1-2s'_X)}, 1\right]$
Case 4: low A-language status and stronger assimilationism $0 < s_X < \alpha(1+\alpha)^{-1}$ and $s'_X > 1/2$	$(0, y, 1-y),\ \forall y \in \left(\frac{2s'_X-1}{2s'_X-s_X}, 1\right]$
Case 5: intermediate A-lang. status and stronger assimilat. $\alpha(1+\alpha)^{-1} < s_X < (1+\beta)^{-1}$ and $s'_X > 1/2$	$(1, 0, 0) \vee (0, y, 1-y),\ \forall y \in \left(\frac{2s'_X-1}{2s'_X-s_X}, 1\right]$
Case 6: high A-language status and stronger assimilationism $(1+\beta)^{-1} < s_X < 1$ and $s'_X > 1/2$	$(1, 0, 0)$

set $(0, 0, w)$, the eigenvalues are $\lambda_i = \left\{(s_X - (1-s_X)\alpha)(1-w) - (1-2s'_X)w;\ (1-s_X-\beta s_X)(1-w);\ 0\right\}$, thus indicating a neutrally stable set if $s_X > (1+\beta)^{-1}$ and $s'_X < 1/2$ are both satisfied. In the latter case, the neutrally stable set satisfies $(0, 0, w) \therefore \frac{s_X-(1-s_X)\alpha}{(s_X-(1-s_X)\alpha)+(1-2s'_X)} < w \leq 1$. The latter corresponds to Case 3 in Table 1. At the set $(0, y, 1-y)$, the eigenvalues are $\lambda_i = \left\{-(1-s_X)y - (1-2s'_X)(1-y);\ -(1-s_X-\beta s_X)y;\ 0\right\}$. The latter indicates that $(0, y, 1-y),\ \forall y \in [0, 1]$ is a neutrally stable set if both $s_X < (1+\beta)^{-1}$ and $s'_X < 1/2$ are satisfied, corresponding to Cases 1 and 2 in Table 1. On the other hand, if $s_X < (1+\beta)^{-1}$ and $s'_X > 1/2$ are both satisfied, then $(0, y, 1-y)$ is a neutrally stable set if $\frac{2s'_X-1}{2s'_X-s_X} < y \leq 1$, corresponding to Cases 4 and 5 in Table 1. Although the system is non-hyperbolic at the sets $(0, y, 1-y)$ and $(0, 0, w)$ and the analysis of the eigenvalues of θ is not sufficient to guarantee stability at those sets, the results in Table 1 do match the dynamics and topology in the interior of Ω and can be confirmed from the numerical simulations to be presented in Section 3. ■

3. Discussion

In this section, I state three propositions based on the proof of Proposition 1 and discuss the possible social outcomes in the long run equilibria.

Proposition 2 (*Two Possible Social Outcomes*). *When the status of language A is intermediate and the status of the assimilationist ideology is stronger than the nationalistic one, the population of the country either evolves to a monolingual group of A-language speakers in which individuals speaking B get assimilated and nationalism disappears, or to a state composed of two segregated groups, one of nationalistic individuals, all speaking language A, and the other of monolinguals speaking language B. In the latter case, linguistic segregation and isolation are the negative social outcomes of the interplay between ideological struggle and language competition.*

Proof. This corresponds to Case 5 in Proposition 1. ■

Figs. 2 and 3 present respectively the phase space and the time evolution for two simulations using the following parameters: $s_X = 0.575$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$, i.e., a country in which the assimilationist ideology status is one and a half times stronger than the nationalistic ideology status, language A has an intermediate status and bilinguals are moderately seen as representatives of language A for the monolinguals in Y but individuals in X weakly see bilinguals as representatives of language B. With initial conditions $(x_0, y_0, w_0) = (0.05, 0.05, 0.8)$ (left panels), the share of group X is monotonically increasing due to the combined effect of language transitions $Z \to X$ and ideology transitions $W \to X$. At time $t = 0$, both assimilationists speaking language A and monolinguals of language B represent each only 5% of the total population. Due to $\beta \gg \alpha$, together with the higher status of language A, monolinguals in Y have a stronger incentive to become bilinguals further becoming fully assimilated over time, i.e., $Y \to Z \to X$, until no monolingual of language B remains in the population. Without the latter, bilinguals also disappear in the long run. At the same time, regarding ideology struggle, nationalists shift their ideology due to $s'_X > s_W$ and the rate of shift increases over time as the share of individuals belonging to X increases, until no nationalist remains in the population. The simulation displayed in the left panels of Figs. 2 and 3 can be seen as a social outcome in which B-language speakers become linguistically and culturally assimilated if one follows [18] and adopts language as a proxy for culture.

On the other hand, with initial conditions $(x_0, y_0, w_0) = (0.1, 0.7, 0.1)$ (right panels), one can see the evolution towards a segregated country with two monolingual groups and nationalism surviving in the long run. In terms of language transition, although monolinguals of language B still have an incentive to shift to bilingualism due to the higher status of language A and $\beta \gg \alpha$, in this second simulation the share of individuals in Y is very large at $t = 0$ (70%), also making shifts from X to Z

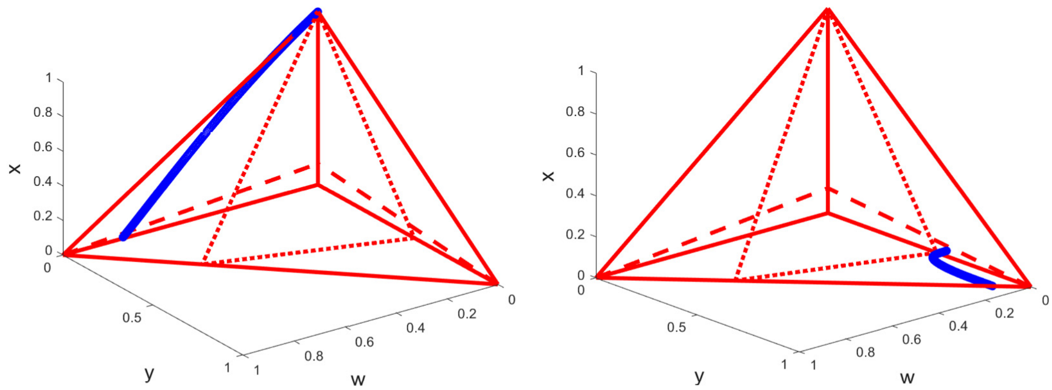

Fig. 2. Phase spaces for parameters: $s_X = 0.575$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions $(0.05, 0.05, 0.8)$ (left panel) and $(0.1, 0.7, 0.1)$ (right panel); dotted (dashed) lines represent plane $\dot{x} = 0$ ($\dot{y} = 0$); step size $\Delta t = 0.05$ (colour online).

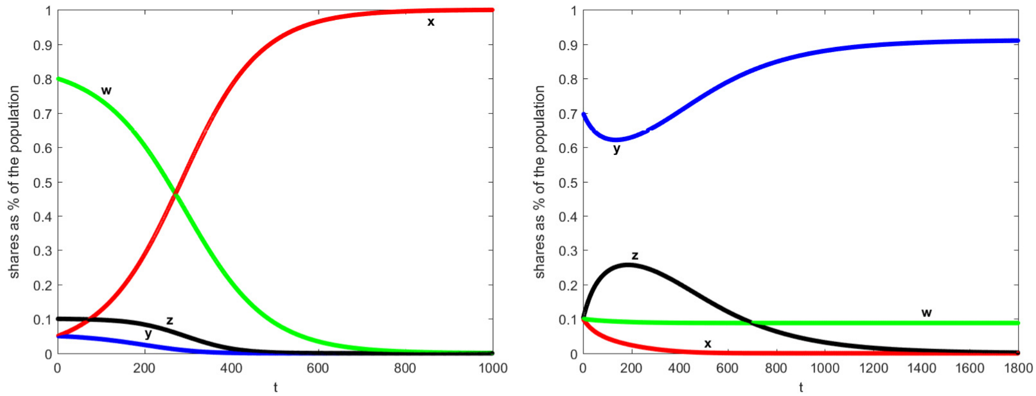

Fig. 3. Time evolution for parameters: $s_X = 0.575$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions $(0.05, 0.05, 0.8)$ (left panel) and $(0.1, 0.7, 0.1)$ (right panel); step size $\Delta t = 0.05$; red: assimilationists; green: nationalists; blue: B-language monolinguals; black: bilinguals. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

attractive. The share of group X decreases until disappearing from the population while the share of monolinguals of language B decreases up to $t \approx 150$ but then, with the extinction of group X on the way, the incentive to learn language A ceases and bilinguals shift from Z to Y leading to an increase of the latter group. Regarding ideology struggle, as in the first simulation, nationalism is monotonically decreasing, but once assimilationist individuals in X disappear, the share of nationalists that remains is no more influenced by those to shift their ideology and A-language monolinguals remain nationalist, sharing the same country with B-language monolinguals. Such an equilibrium might lead to radicalism and violence between the two different cultural groups. In the simulation displayed in the right panels of Figs. 2 and 3, such outcome results even with a small share of 10% of nationalists at the initial conditions and with the status of the assimilationist ideology being 1.5 times stronger than that of nationalism. Thus, an outcome in which nationalism takes over the heterogeneous population can occur even when the assimilationist status is stronger. In contrast, in the first simulation displayed in the left panels of the same Figures, nationalism disappeared despite being supported by 80% of the population at $t = 0$.

Proposition 3 (*Segregated Society*)**.** *When the status of nationalism is stronger than assimilationism, the population always evolves in the long run to an equilibrium with segregation in which nationalists become isolated from any surviving group. On the other hand, when assimilationism is stronger, segregation is still feasible unless the A-language has a high status. Bilingualism can be sustained only in a segregated society when nationalism is the strongest ideology and the status of the language spoken by nationalists is high.*

Proof. This corresponds to Cases 1–4 in Proposition 1 and to the equilibrium corresponding to the neutrally stable set at $(0, y, 1 - y)$ in Case 5. ■

Fig. 4 presents the phase space and time evolution for a simulation of Case 1, when the A-language status is low and nationalism is the strongest ideology, using parameters $s_X = 0.1$; $s'_X = 0.3$; $\alpha = 0.2$; $\beta = 0.6$. In this case, $\mathcal{P}_X \bigcap \Omega_{\text{interior}} = \emptyset$ such that $\dot{x} < 0$ at any point in the interior of the phase space. Also, the initial condition $(x_0, y_0, w_0) = (0.9, 0.08, 0.02)$ is

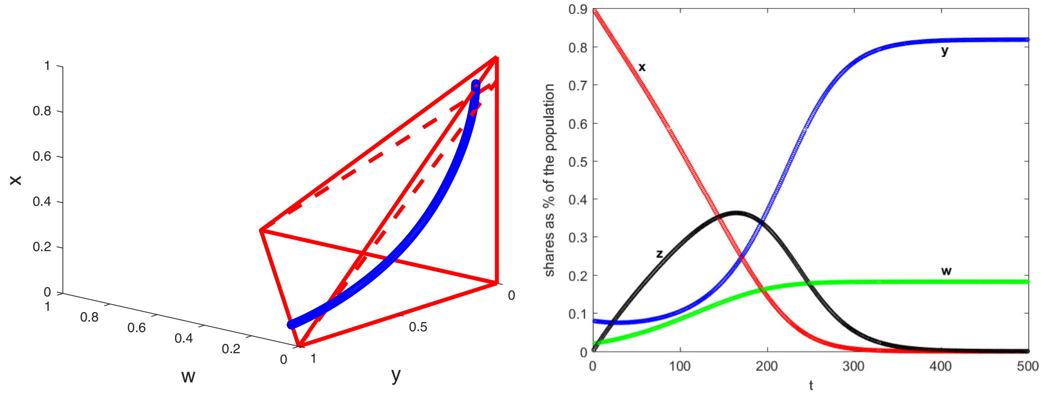

Fig. 4. Phase space and time evolution for parameters: $s_X = 0.1$; $s'_X = 0.3$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions (0.9, 0.08, 0.02); dashed lines represent plane $\dot{y} = 0$; step size $\Delta t = 0.05$; right panel: assimilationists (red); nationalists (green); *B*-language monolinguals (blue); bilinguals (black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

located above $\mathcal{P}_Y$ such that initially group Y decreases with individuals learning language A despite the low status of the latter. This happens due to $\beta \gg \alpha$ and the large size of group X (initially 90% of the total population). The latter group is monotonically decreasing mainly due to language transitions $X \to Z$ (low status of language A) and also due to ideology transitions $X \to W$ because of the strength of nationalism. Bilingualism grows with individuals in both monolingual groups learning a second language initially, but with the share of group X quickly decreasing in the total population, the incentive to attract individuals from group Y to bilingualism ceases and the latter group starts to grow significantly after $t = 100$. Bilingualism keeps growing up to $t = 160$ and then decreases monotonically with the effect of vertical transmission of language $Z \to Y$ now being stronger than the effect of horizontal transmission $X \to Z$. After $t = 300$, no individual remains in group X and nationalists in W become isolated from the rest of the population. At $t = 400$, the last bilingual individual disappears from the population and the equilibrium is achieved with $(x^*, y^*, w^*) = (0, 0.818, 0.182)$. In Case 2, the status of language A is intermediate but such an effect is not strong enough to change the type of equilibrium in the phase space. The social outcome in the long run is still segregation as in Case 1, with nationalists and *B*-language monolinguals surviving.

Fig. 5 is a simulation of Case 4 and uses the same parameters as in Case 1 but $s'_X = 0.6$ (nationalism as the weakest ideology instead) and initial condition $(x_0, y_0, w_0) = (0.05, 0.05, 0.88)$, which is located below $\mathcal{P}_Y$ ($\dot{y} > 0$) and above $\mathcal{P}_X$ ($\dot{x} > 0$). Compared to Case 1, the ideological difference in Case 4 is not strong enough to change the equilibrium in the phase space. Now, nationalism is monotonically decreasing due to the strength of the assimilationist ideology among *A*-language speakers. Ideology transitions $W \to X$ feed group X, which initially grows. Bilinguals also grow their share in the population but only due to transitions $X \to Z$, given both the low status of the *A*-language and the small share of group X in the population (the latter as opposed to the simulation for Case 1) are not strong enough to attract *B* monolinguals to learn a second language. Group Y grows but at a very small rate until $t = 400$ when the share of bilinguals is large enough to speed up the growth rate through vertical language transmission. At $t = 400$, group W has more than halved its size, which together with the effect of horizontal language transmission from *A*-monolingualism to bilingualism, leads the share of group X to achieve its peak and start to decrease monotonically (plane $\mathcal{P}_X$ is crossed) until extinction of open-minded *A*-monolinguals at $t = 700$. As in Case 1, when the latter event takes place, nationalists become isolated from the rest of the population and later at $t = 800$ bilinguals disappear due to transitions $Z \to Y$.

Regarding Case 3, a different kind of segregation takes place when compared to the other cases. Fig. 6 displays two simulations with parameters $s_X = 0.75$; $s'_X = 0.3$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions (0.20, 0.15, 0.60) (left panel) and (0.05, 0.05, 0.05) (right panel). In both cases, given $s_X > (1+\beta)^{-1}$, $\mathcal{P}_Y \bigcap \Omega_{\text{interior}} = \emptyset$ such that $\dot{y} < 0$ at any point located in the interior of the phase space. Due to the high status of language A, group Y is monotonically decreasing through horizontal language transmission, i.e., transitions $Y \to Z$. In the left panel, the initial condition is located above $\mathcal{P}_X$ such that $\dot{x} < 0$. Group X is monotonically decreasing due to ideology transitions $X \to W$. Such transitions are not compensated by vertical language transitions $Z \to X$ and open-minded *A*-language monolinguals become extinct. Once group X disappears at $t = 250$, the only transitions that keep taking place in the population are $Y \to Z$ until *B*-monolinguals become extinct. The latter is a consequence of the high status of language A and the fact that bilinguals are moderately seen as representatives of language A for the monolinguals in Y ($\beta = 0.6$). In the long run, only *A*-monolingual nationalists and bilinguals survive. Bilinguals in this case are basically composed of former *B*-language monolingual individuals, therefore segregation in Case 3 can still lead to violence as in the other cases, despite bilingual individuals being able to speak the language of nationalists. In such a case, bilingualism might not be a desired social outcome as generally found in the literature on language competition.

Although neutral stability generally implies segregation in equilibria located at either $(0, y, 1-y)$ or $(0, 0, w)$, it should be pointed out that complete assimilation with only *B*-monolinguals surviving (equilibrium at (0, 1, 0)) or, instead, nationalists taking over the entire population (equilibrium at (0, 0, 1)) can occur as very particular cases of neutral stability. On the

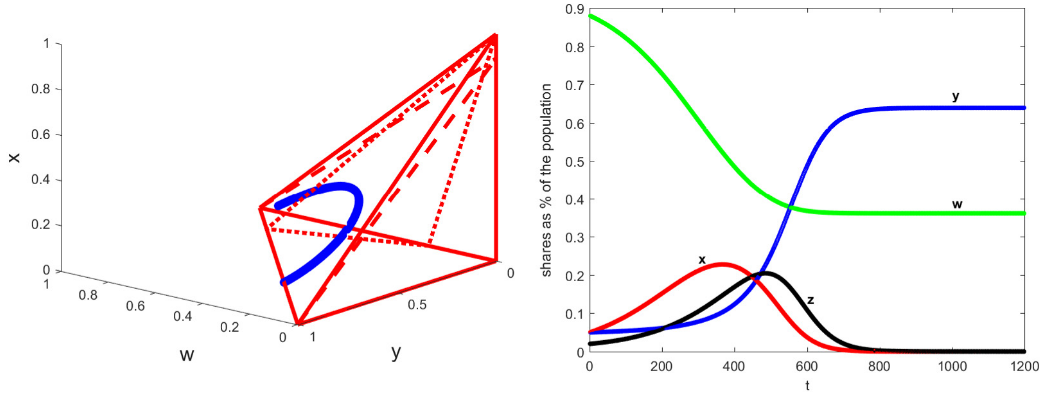

Fig. 5. Phase space and time evolution for parameters: $s_X = 0.1$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions $(0.05, 0.05, 0.88)$; dotted (dashed) lines represent plane $\dot{x} = 0$ ($\dot{y} = 0$); step size $\Delta t = 0.05$; right panel: assimilationists (red); nationalists (green); B-language monolinguals (blue); bilinguals (black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

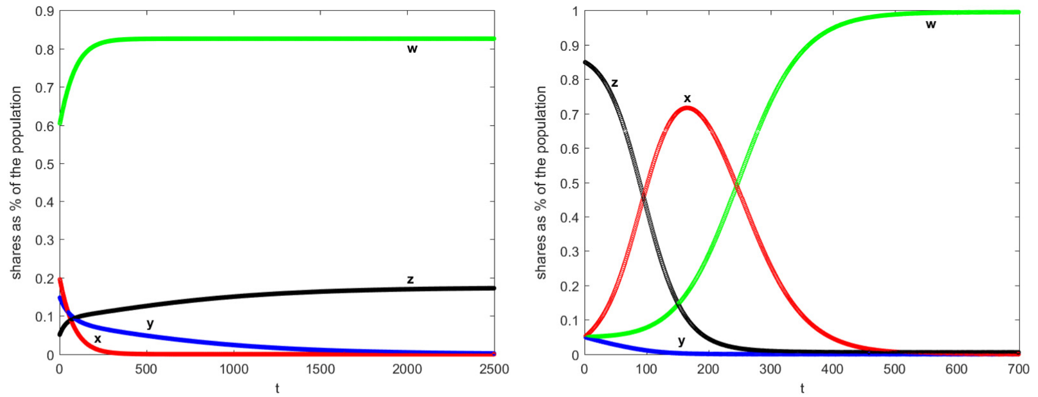

Fig. 6. Time evolution for parameters: $s_X = 0.75$; $s'_X = 0.3$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions $(0.20, 0.15, 0.60)$ (left panel) and $(0.05, 0.05, 0.05)$ (right panel); step size $\Delta t = 0.05$; assimilationists (red); nationalists (green); B-language monolinguals (blue); bilinguals (black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

other hand, such equilibria are not robust when compared to the case of complete assimilation at the asymptotically stable state $(1, 0, 0)$. Due to neutral stability, any disturbance of the dynamic system (some individuals changing their ideology or even a group of speakers of the competing language immigrating to the country) is likely to move the equilibrium to another closely located point at the neutrally stable set where segregation is re-established. For the sake of completeness, an example of neutrally stable equilibrium in which only nationalists survive is shown in the right panel of Fig. 6. As in the left panel, nationalists grow monotonically while group Y is monotonically decreasing until extinction at $t = 150$. But at the initial condition nationalists are just a small share while bilinguals are 85% of the total population. Because of the latter, differently from the left panel, vertical transmission of language $Z \to X$ is now able to compensate ideology transitions $X \to W$ and up to $t = 200$ group X increases (population state located below $\mathcal{P}_X$, i.e., $\dot{x} > 0$). With bilingualism quickly decreasing due to a larger group X and the high status of the A-language, together with the size of group W growing and starting to become significant, open-minded A-monolinguals start to decrease after $t = 200$. The overall effect of both opposing forces is that now transitions $Z \to X$ are no more able to compensate transitions $X \to W$. Bilinguals disappear from the population followed by the extinction of A-monolinguals in group X and only nationalists survive.

Finally, segregation in Case 5 was already addressed in Proposition 2.

Proposition 4 (*Complete Assimilation*)**.** *A robust (asymptotically stable) equilibrium with complete language assimilation implies the survival of the language spoken by nationalists. Moreover, an equilibrium in which such a social outcome does not rely on the initial conditions requires not only assimilationism to be the strongest ideology, but also a sufficiently high status of the surviving language in order to eliminate all speakers of the competing language in the long run with certainty.*

Proof. Complete assimilation with certainty corresponds to Case 6 in Proposition 1 and complete assimilation depending on the initial conditions corresponds to the equilibrium at the asymptotically stable fixed point $(1, 0, 0)$ in Case 5. ■

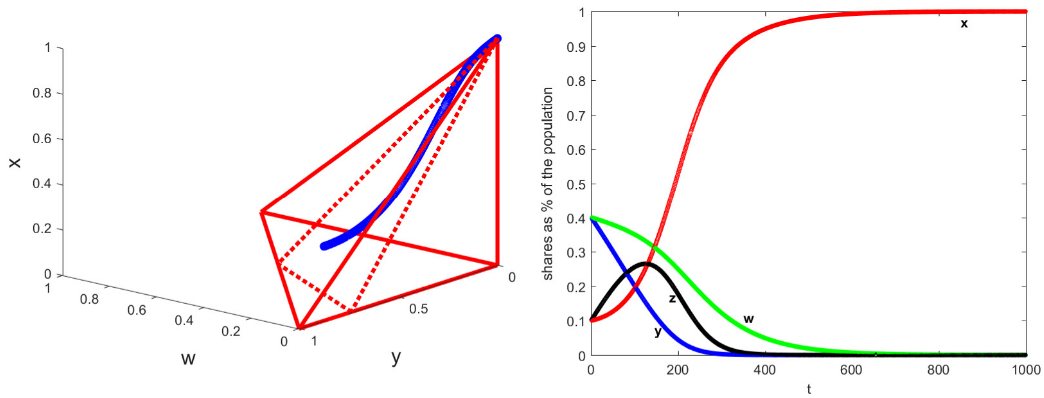

Fig. 7. Phase space and time evolution for parameters: $s_X = 0.75$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions (0.10, 0.40, 0.40); dotted lines represent plane $\dot{x} = 0$; step size $\Delta t = 0.05$; right panel: assimilationists (red); nationalists (green); B-language monolinguals (blue); bilinguals (black). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In Case 6, due to the high status of language A and a stronger assimilationist ideology, both y and w are monotonically decreasing given that $(1-s_X-\beta s_X)(1-x-y-w)-xs_X < 0$ and $(1-2s'_X) < 0$ in (8) and (9), respectively. On the other hand, $(s_X - \alpha(1-s_X))(1-x-y-w)-(1-s_X)y-(1-2s'_X)w$ in (7) is strictly negative (resp. positive) if (x, y, w) is located above (resp. below) $\mathcal{P}_X$. Thus, independently of the initial conditions, with the share of nationalists and B-language monolinguals decreasing over time, any trajectory in the interior of the phase space will always reach or already start at a population state located below $\mathcal{P}_X$, in which A-language monolinguals in group X will continue to increase monotonically until converging to the global attractor $(1, 0, 0)$. Fig. 7 displays a simulation for Case 6 with: $s_X = 0.75$; $s'_X = 0.6$; $\alpha = 0.2$; $\beta = 0.6$; initial conditions $(x_0, y_0, w_0) = (0.1, 0.4, 0.4)$. At time $t = 0$, group X is already increasing due to transitions $W \to X$ and $Y \to Z \to X$. A weaker nationalistic ideology together with the high status of language A and $\beta \gg \alpha$ are enough to create the incentive for attracting B-language speakers to bilingualism and further to A-language monolingualism, as well as to lead to the extinction of nationalism in the long run. Complete assimilation in Case 5 was already addressed in Proposition 2.

4. Effect of different transition coefficients

In this section, I extend the analysis of the model to study the effect of different transition coefficients c_i in the original system defined in (1)–(3). I analyse the particular case when the transition coefficient from the nationalistic share to the more open-minded share of the population speaking language A is smaller than the other transition coefficients, i.e., $c_{WX} = \gamma c_i = \gamma c,\ \gamma \in (0, 1)$. In this case, the system in (1)–(3), after making the same assumptions regarding τ and the time interval dt as in Section 2, becomes:

$$\dot{x} = x\left[(s_X - \alpha(1-s_X))(1-x-y-w)-(1-s_X)y-(1-s'_X-\gamma s'_X)w\right] \tag{10}$$

$$\dot{y} = y\left[(1-s_X-\beta s_X)(1-x-y-w)-xs_X\right] \tag{11}$$

$$\dot{w} = xw(1-s'_X-\gamma s'_X). \tag{12}$$

The following proposition shows that, even in this case where nationalists are less likely to change their ideology when compared to assimilationists being attracted to nationalism, there is no robust equilibrium in which nationalism drives all other groups to extinction.

Proposition 5. *When the transition coefficient from the nationalistic share to the more open-minded share of the population speaking language A is smaller than the other transition coefficients, the likelihood of an equilibrium with nationalism and segregation increases but there is no asymptotically stable state in which nationalists take over the entire population.*

Proof. The fixed points of the systems given by (1)–(3) and (10)–(12) are the same, including the sets of fixed points $(0, y, 1-y)$ and $(0, 0, w)$. Thus, there is no asymptotically stable state at $(0, 0, 1)$, in line with Section 2. But now, (i) nationalism is more difficult to defeat as an ideology given that $dw/dt < 0$ only if $1/2 < (1+\gamma)^{-1} < s'_X < 1$, i.e., the assimilationist ideology has to be even stronger than in Section 2 such that the share of nationalists starts to decrease over time; (ii) the fixed point $(1, 0, 0)$, associated with complete assimilation, is asymptotically stable only if the A-language status is intermediate or high, as before, but if $s_X{}' > (1+\gamma)^{-1}$ is also satisfied, thus for a smaller subset of the parameters space; (iii) while there is neutral stability at the set of fixed points $(0, y, 1-y)$ for $s_X < (1+\beta)^{-1}$ and $\forall s'_X$, as in Section 2, neutral stability at the set of fixed points $(0, 0, w)$ now requires both $s_X > (1+\beta)^{-1}$ and $s_X{}' < (1+\gamma)^{-1}$ to be satisfied, thus equilibria with nationalism and segregation of the population in the country now exists for a wider subset of the parameters space; (iv) moreover, whenever $(1, 0, 0)$ is not the only possible equilibrium (Case 5 in Table 1), the set of neutrally stable fixed points at $(0, y, 1-y)$ is wider than before, given the projection of the latter set on the y-axis of the phase space is now given by the set $\left(\frac{(\gamma+1)s'_X-1}{(\gamma+1)s'_X-s_X}, 1\right]$, which is wider than the projected set $\left(\frac{2s'_X-1}{2s'_X-s_X}, 1\right]$ when $\gamma = 1$ as in Section 2. ■

5. Conclusion and further research

The paper contributed to the existing literature on language competition by studying the interplay between language competition and ideology struggle in a mean-field model assuming homogeneous mixing. Among the six possible cases studied, depending on the status of the nationalistic ideology and on the status of the language spoken by nationalists, in four cases, social segregation and survival of nationalism were certain outcomes in the long run, while there was still some likelihood of happening in a fifth case. Even when an assimilationist ideology is relatively stronger, nationalism might survive in equilibrium leading to social segregation. In such a case, nationalism and the isolation of the nationalistic community might pave the way to future radicalization. Complete assimilation and the extinction of nationalism is also possible and implies the survival of the language spoken by former nationalists.

Further research could try to fit the model to real world data. Although theoretical-empirical studies with successful attempts to estimate parameters and apply dynamic system models to real world cases can be seen in [4,7], among others, this paper focused only on the theoretical side in order to provide new useful information by explaining what happens when ideology has an effect on language competition. As pointed out in [2]: "Mathematical models can provide principles rather than numbers. An example of this is seen in the predator–prey model originally developed by Lotka and Volterra. There are few cases where the model fits well with real data, but it does furnish the principle, called Volterra's principle, that moderate harvesting across both species will cause the numbers of the prey species to rise [22]. The principle is well observed in the fishing industry and in crop-spraying programmes, without actual data being fitted to the model".

Despite not looking at the empirical side, the results of the model are qualitatively in line with real world situations. A recent example of the interplay between ideology and language took place in Dade County, Greater Miami, USA, from the 1960s until the early 1990s and is described in Ref. [23]. After the Cuban Revolution, Cuban migration to the USA, which main destination was South Florida, took place initially in three waves, Jan/1959–Oct/1962, Sep/1965–Apr/1966 (freedom flights) and Apr–Oct/1980 (Mariel boatlift), respectively. The first two waves included skilled workers and high officials of the deposed government, which together with the Cold War, was an incentive for American natives to welcome those immigrants. As a result, in 1973, Dade County even declared itself officially bi-cultural. However, the third wave was composed of a large amount of working class immigrants, many of them black, triggering nationalism, anti-Cuban demonstrations and African American riots in South Florida. Those events culminated with voters repealing the bi-cultural status and Dade County declaring that it was unlawful the use of public funds to promote any language other than English or any culture other than that of the USA. Despite the third wave impact after 1980, the situation between 1959–1980 could be modelled as in Case 6: English language status would be high, $s_X > (1+\beta)^{-1}$, and pro-immigration would be the strongest ideology, $s'_X > 1/2$, the latter being even officially supported by Dade County government. A large group X composed of native Americans supporting Cuban immigration would be a strong incentive for Cuban immigrants to become bilingual overtime and raise their children as either bilinguals or English monolinguals. In the absence of new events such as the third wave, complete assimilation would take place in the long run as in the typical US three-generations pattern of complete assimilation of immigrants discussed in [20,21].

Another possibility regarding further research would be to extend the Lotka–Volterra-type model in Eqs. (1)–(3). A combined predator–prey-epidemic model could be studied. As a first attempt, language competition would remain a purely predator–prey model as in this paper while ideology struggle could follow an epidemic process. A-language monolinguals could be part of the two groups in this paper, X and W, plus a third new group named R. Individuals in group X would be open-minded young individuals born to either A-monolingual or bilingual open-minded parents. These open-minded A-language speakers might either remain in X when they become adults or learn a second language and shift to Z. Ideologically speaking, they would be susceptible individuals who could join a nationalistic community due to contact with the "infected" radical members of group W. Differently from the model in this paper, the latter would not shift back to X. Instead, over time individuals in W could "lose faith" on the nationalistic ideology and leave the nationalistic community, shifting to a new group R of removed individuals similar to the hardened unbelievers in [3]. Individuals joining group R would not be willing to interact with anyone from group Z, such that they could be considered similar to individuals without a well defined ideology, i.e., neither nationalists nor assimilationist individuals. Finally, over time individuals in group R could remain in R or become "open to reconversion", i.e., shift back to X and become again open-minded and susceptible to rejoin nationalism in the future ($X \to W$) or not (either $X \to X$ or $X \to Z$).

Acknowledgements

I acknowledge the research support "Programa de Incentivo à Produtividade em Ensino e Pesquisa" from PUC-Rio. I thank the referees for their insightful suggestions and comments, which helped me to improve the manuscript.

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